FIELD LOCALIZATION ON A BRANE INTERSECTION IN ADS SPACETIME

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INTRODUCTION

Branes

the most important ingredients to construct higher-dimensional cosmological and particle physics models

4D spectrum (GR+SM) from higher-dimensions

Compactification

a traditional scheme

4D effective theory is obtained as the result of spectral decomposition

= 4D mode + higher-dimensional modes

mass of higher modes ~1/(size of extra space)

no evidence of extra-dimensions

⇒ Extra dimensions must be very small

Presence of the branes

Branes

: submanifolds embedded into higher-dimensional spacetime

: give boundary conditions for the field propagating into the extra space

Special choice of BC leads to *localization* of the higher-dimensional fields on a brane.

⇒ 4D theory is locally reproduced

⇓

allowing large or non-compact extra-dimensions

new type of singnatures

Braneworld

Large extra dimensions Arkani-Hamed, Dimopoulos and Dvali(1998)

compact extra space

SM particle : confined on the boundary branes

Gravity: propagate into whole spacetime

Undemocratic treatment of gravity and SM

may lead to resolution of hierarchy problem

L size of extra-dimensions

M_{4+n} (4+n)-dimensional Planck scale

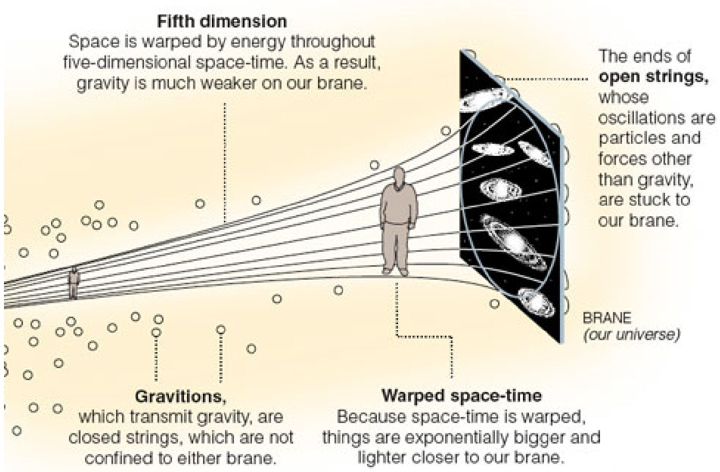
effective 4D Planck mass $M_4^2 = M_{4+n}^{2+n} L^n$

$n = 2 \quad L \sim 0.1mm \Rightarrow M_6 \sim TeV$

does only work in dimensions $D \geq 6$

no brane gravity

Randall-Sundrum



Randall Sundrum (1999)

$$D = 5$$

brane tension curves the extra space

warped structure

gravitons are trapped around the brane

4D gravity is recovered in low energy regime

in spite of the *infinitely* extended extra space

hierarchy problem is not resolved

We will consider a kind of higher-dimensional extensions of RS

INTERSECTING BRANES

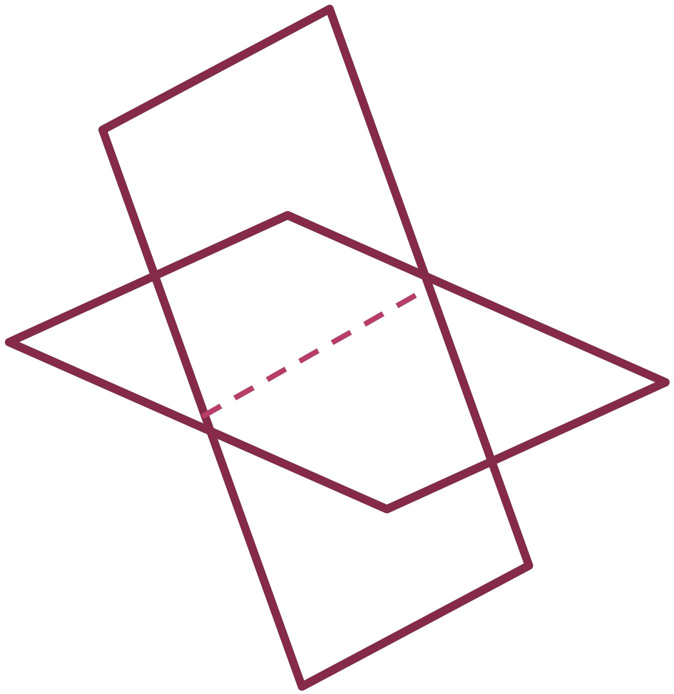
in higher-dimensional spacetime, there would be intersections of branes

Intersecting branes models

may lead to the SM model

mainly considered in flat background

We are going to investigate...



Localizability of fields with various spins which appear in SM

in curved background

with brane self-gravity

INTERSECTING BRANES IN 6D

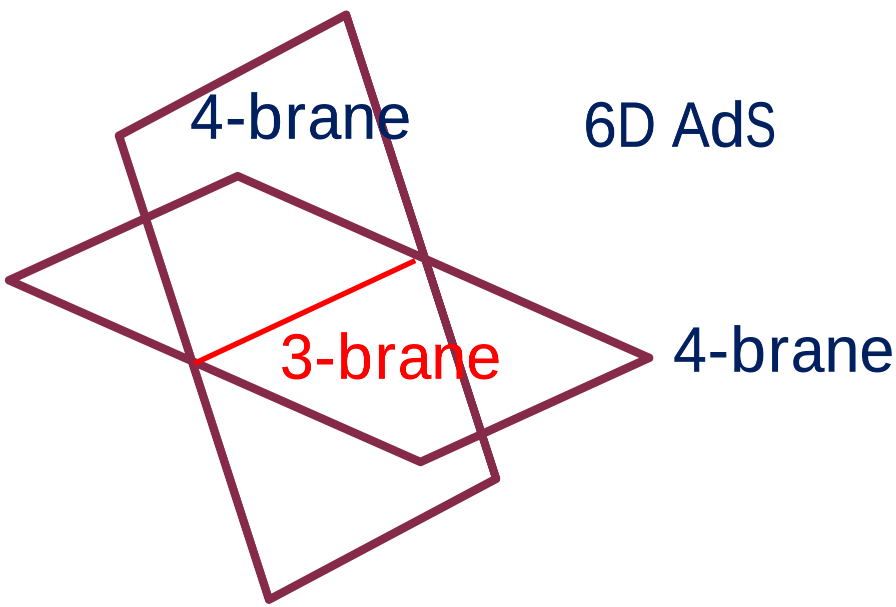
(Kaloper '04)

two intersecting 4-branes and a 3-brane residing on intersection are embedded in 6D AdS

$$S = S_{Bulk} + S_{branes}$$

$$S_{Bulk} = -\int d^6 X \sqrt{-g}\left(\frac{M_6^4}{2} R + \Lambda\right) \qquad \Lambda < 0$$

$$S_{branes} = \sum_{i=1,2} \int_{\Sigma_i} d^5 x \sqrt{-q_i}\,(-\sigma_i) + \int_{\Sigma_0} d^4 x \sqrt{-q_0}\,(-\sigma_0)$$



Intersection with right angle

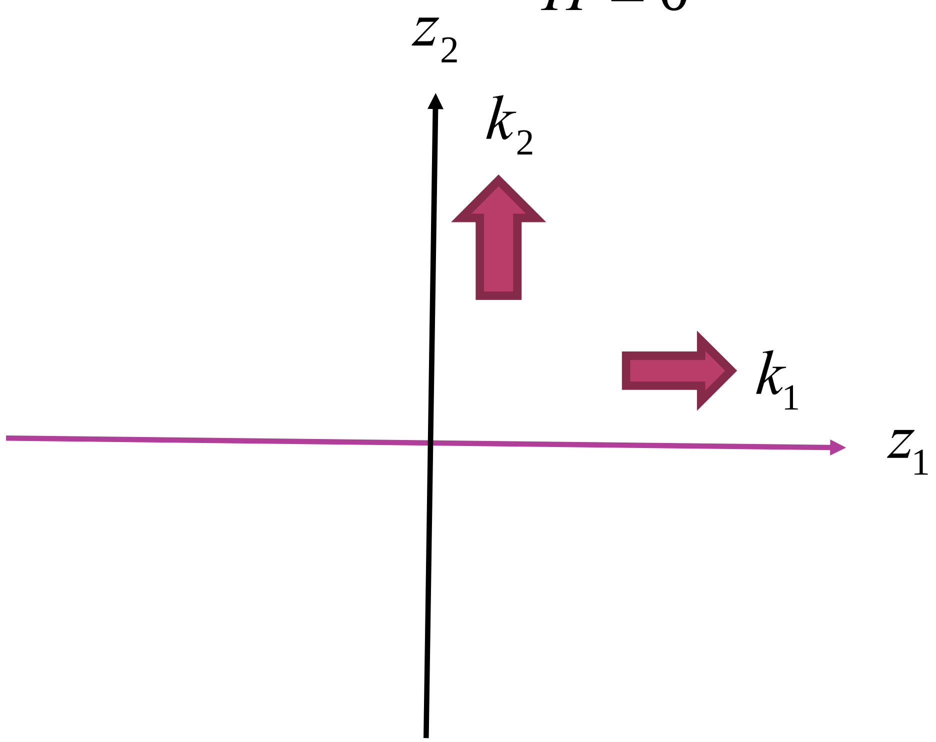
$$ds^2 = A\left(t, z^1, z^2\right)^2 \left(-\delta_{mn} dz^m dz^n + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$

$$A\left(t, z^1, z^2\right) := \left(Ht + k_1 z^1 + k_2 z^2 + C_0\right)^{-1}$$

$$\Lambda = 10(H^2 - k_1^2 - k_2^2)$$

$H \neq 0$ de Sitter

$H = 0$ Minkowski



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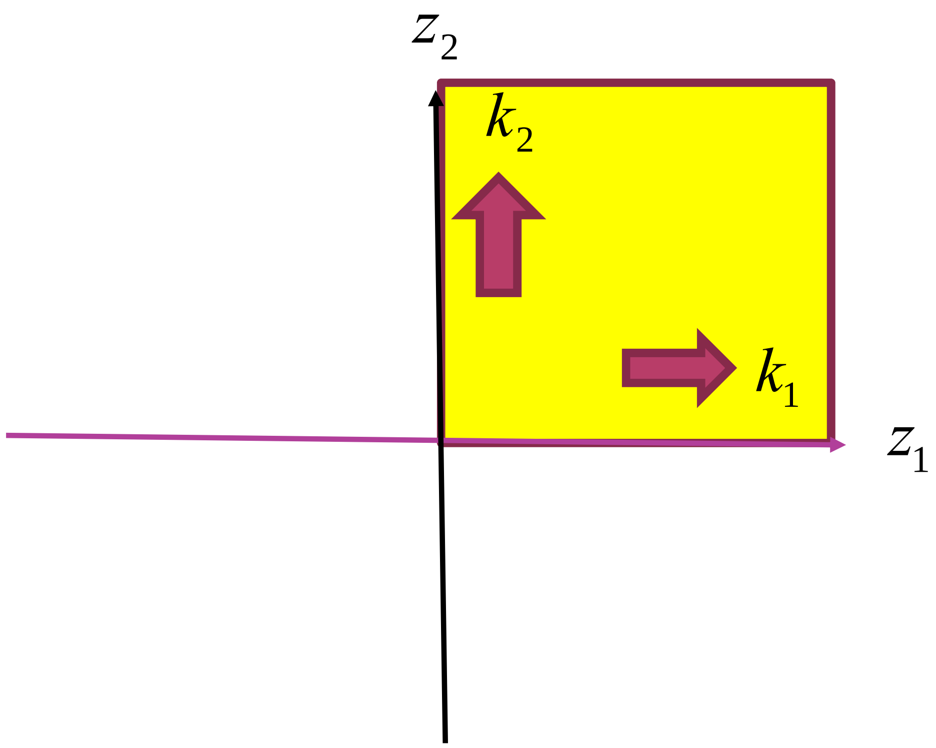


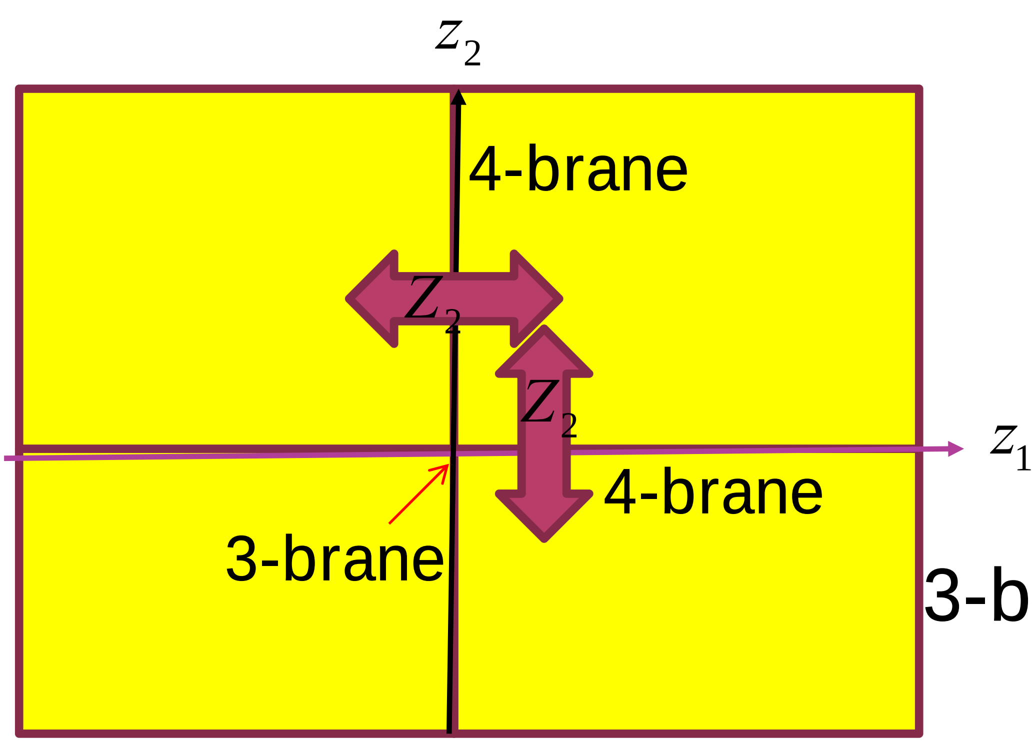
gluing copies of a patch of AdS space

$$ds^2 = A(t, z^1, z^2)^2 \left(-\delta_{mn} dz^m dz^n + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$

$$A(t, z^1, z^2) := \left(Ht + k_1 |z^1| + k_2 |z^2| + C_0\right)^{-1}$$

$$\Lambda = 10(H^2 - k_1^2 - k_2^2)$$

$H = 0$ Minkowski 3-brane



$$\kappa_6^2 \sigma_1 = 8k_1$$

$$\kappa_6^2 \sigma_2 = 8k_2$$

$$\kappa_6^2 \sigma_0 = 0$$

3-brane is tensionless

Intersection with general angles

new coordinate

$$u = \sin\alpha_1 z^1 - \cos\alpha_1 z^2 \quad v = \sin\alpha_2 z^1 - \cos\alpha_2 z^2$$

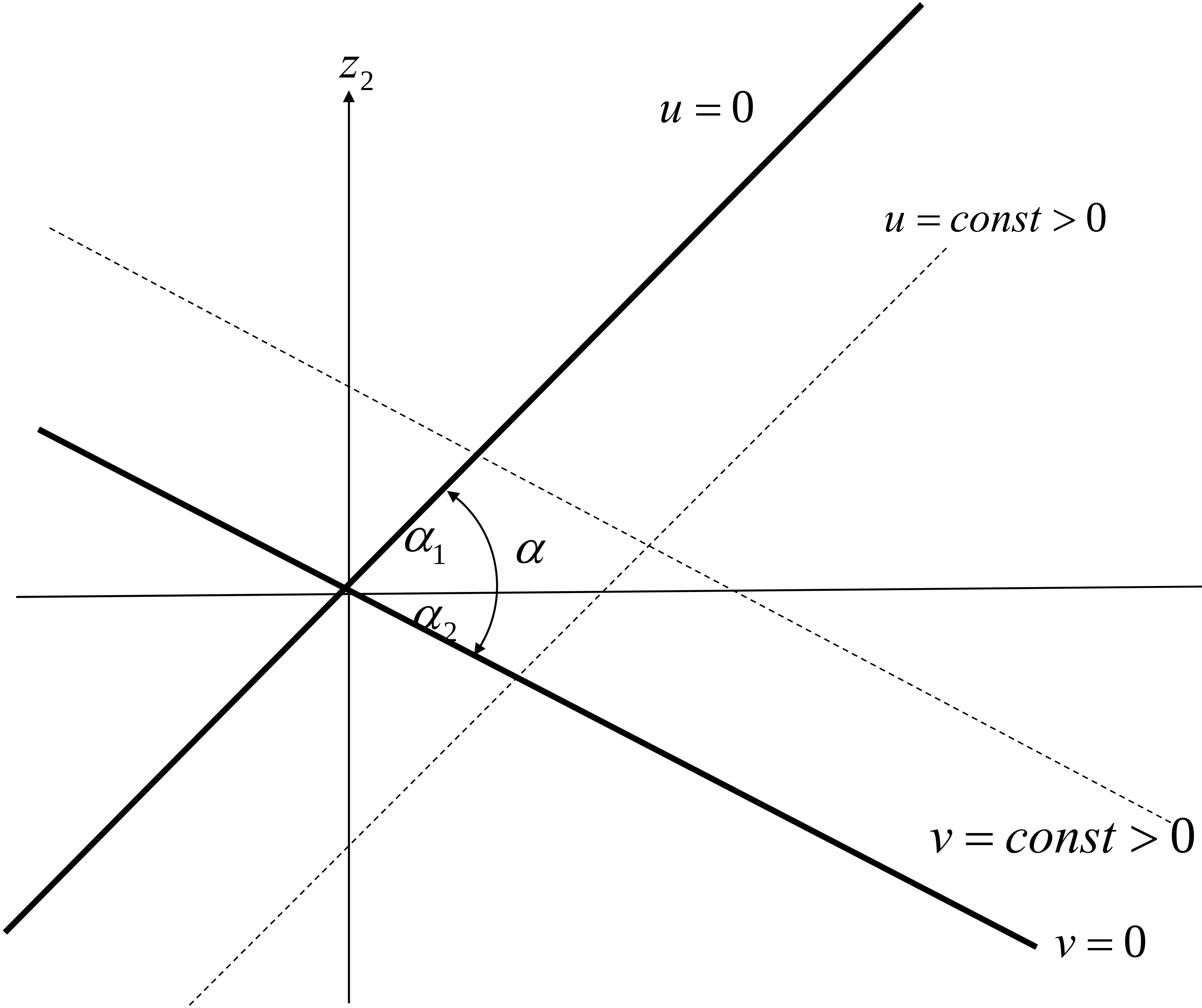
$$ds^2 = A(t,u,v)^2 \left(-\gamma_{mn} d\tilde{z}^m d\tilde{z}^n + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$

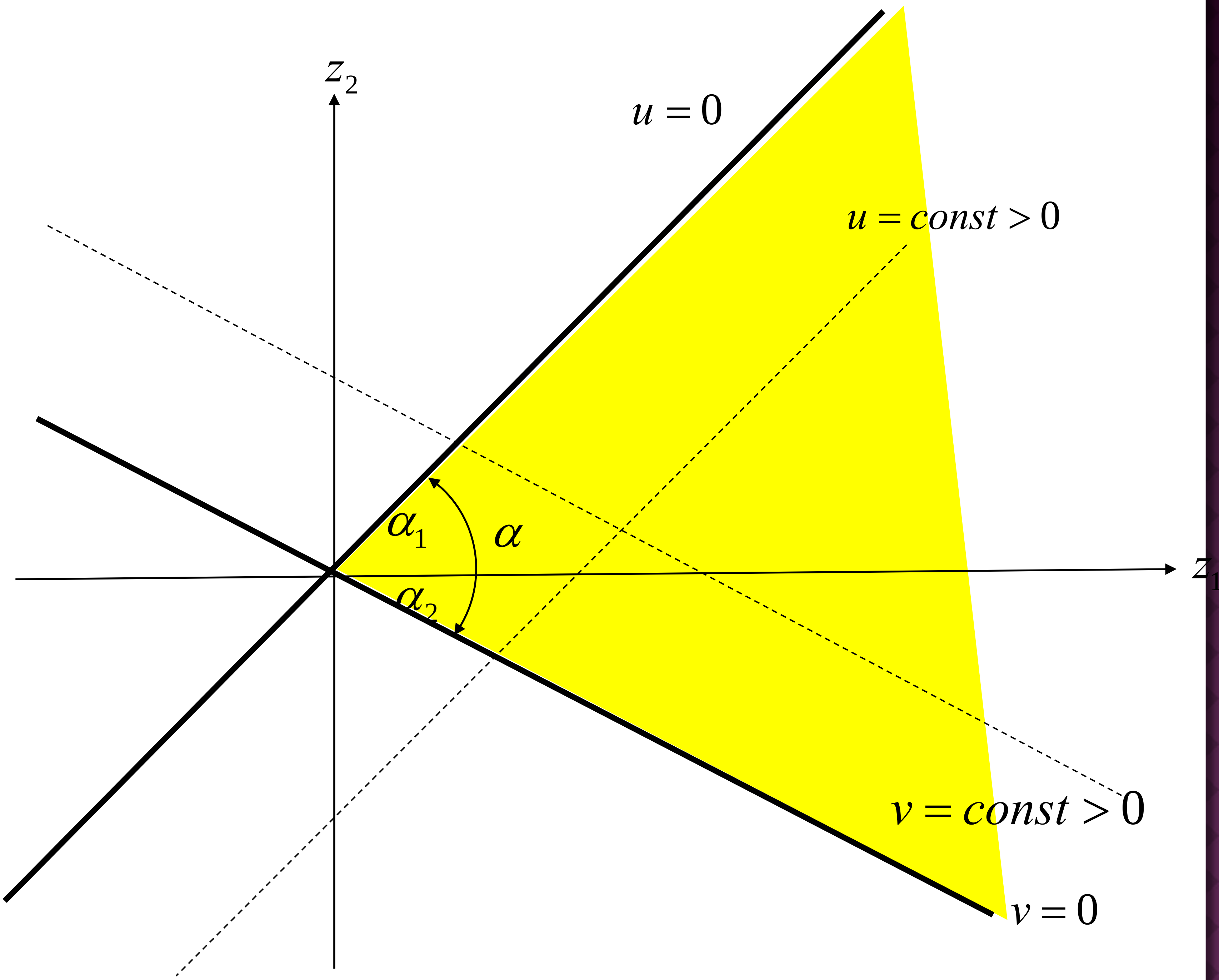
$$C_1 = \frac{k_1 \cos\alpha_2 - k_2 \sin\alpha_2}{\sin(\alpha_1 + \alpha_2)} \quad C_2 = \frac{k_1 \cos\alpha_1 + k_2 \sin\alpha_1}{\sin(\alpha_1 + \alpha_2)}$$

$$A(u,v) := \left(C_1 u + C_2 v + C_0\right)^{-1}$$

4-branes are at $u = 0$ and $v = 0$

A 3-brane is on the intersection $u = v = 0$


z_2
$u = 0$
$u = const > 0$
α_1
α
z_1
α_2
$v = const > 0$
$v = 0$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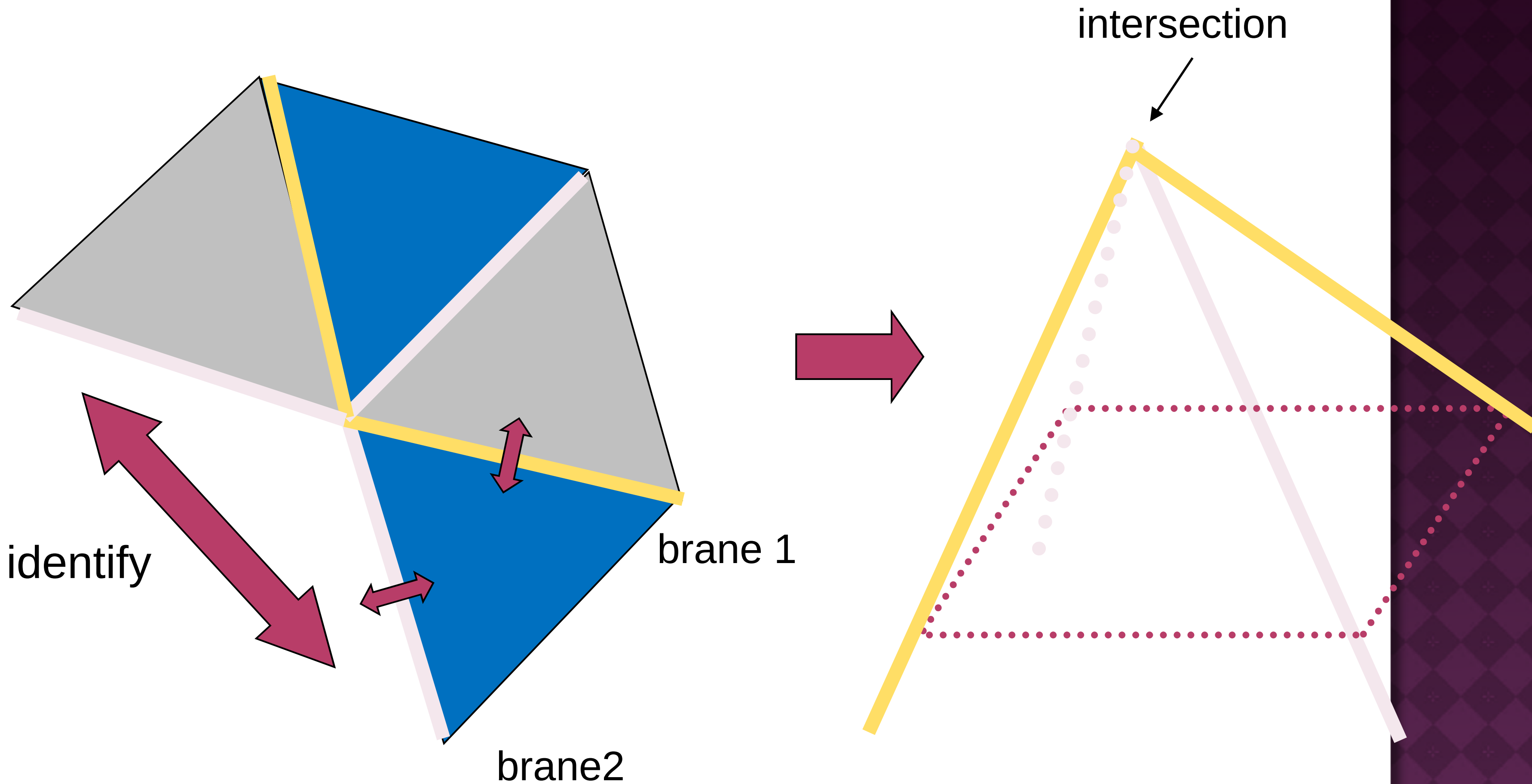

Brane tensions

$$\kappa_6^2 \sigma_1 = 8\left(C_1 - C_2 \cos\alpha\right)$$

$$\kappa_6^2 \sigma_2 = 8\left(C_2 - C_1 \cos\alpha\right)$$

$$\kappa_6^2 \sigma_0 = 4\left(\frac{\pi}{2} - \alpha\right)$$

$$\alpha := \alpha_1 + \alpha_2$$

Intersecting brane solution in 6D AdS

$u \to |u| \quad v \to |v|$

$$ds^2 = A(t,u,v)^2 \left(-\gamma_{mn} d\tilde{z}^m d\tilde{z}^n + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$

$$A(u,v) := \left(C_1|u| + C_2|v| + C_0\right)^{-1}$$

$$C_1 = \frac{k_1 \cos\alpha_2 - k_2 \sin\alpha_2}{\sin(\alpha_1 + \alpha_2)} \quad C_2 = \frac{k_1 \cos\alpha_1 + k_2 \sin\alpha_1}{\sin(\alpha_1 + \alpha_2)}$$

4-branes at $u = 0$ and $v = 0$

3-brane at the intersection $u = v = 0$

PROPERTIES

warped AdS

⇒ graviton zero mode is localized at the intersection

reproduction of 4D gravity at the intersection

$$M_4^2 = \frac{2M_6^4}{3} L^2 \sin\alpha$$

$$L^{-2} = (k_1 \cos\alpha_1 + k_2 \sin\alpha_1)(k_1 \cos\alpha_2 - k_2 \sin\alpha_2)$$

$$L \approx 0.1mm \quad \Rightarrow \quad M_6 \approx \mathrm{TeV}$$

$$M_4 \approx 10^{19} GeV$$

TeV scale gravity

Localizability of fields, which appear in SM spectrum, is going to be investigated.

FIELD LOCALIZATION

The localization of fields of various spins will be investigated

$$S = \int d^6X \sqrt{-g} \; \Big[\; -\frac{1}{4} F_{MN}^2 + \left((M_A^2 - \chi R) g^{MN} - \tau R^{MN} \right) A_M A_N + \frac{1}{2} \left((\partial\phi)^2 - \left(M_s^2 - \xi R \right) \phi^2 \right)$$
$$+ \; i \bar{\Psi} \left(\underline{\gamma}^M D_M + M_f \right) \Psi \Big].$$

M_S scalar mass

$F_{MN} = \partial_M A_N - \partial_M A_N$ M_A mass of the vector field

$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ Dirac spinor (8 components) (ψ_i 4D Dirac spinors)

M_f fermion mass matrix (8×8)

$D_M = \partial_M + \Gamma_M$ covariant derivatives

$\chi \; \tau \; \xi$ coupling to curvature

SCALAR SECTOR

$$S = \int d^6 X \sqrt{-g}\left((\partial\phi)^2 - \left(M_S^2 - \xi R\right)\phi^2 \right)$$

Equation of motion $\Phi = \left(C_1|u| + C_2|v| + C_0\right)^{-2}\phi$

$$\left[-\Box + \frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} - \frac{2\cos\alpha}{s(u)s(v)}\frac{\partial^2}{\partial u\partial v} - 2\cos\alpha\delta(u)s(v)\partial_v - 2\cos\alpha\delta(v)s(u)\partial_u - V(u,v)\right]\Phi = 0$$

$$V(u,v) = \left(M_s^2 + 30\left(\frac{1}{5} - \xi\right)\left(k_1^2 + k_2^2\right)\right)A(u,v)^2 - h_1(u,v)\delta(u) - h_2(u,v)\delta(v)$$

Mode expansions $\Phi = f_0(u,v)\varphi_0(x^\mu) + \int d\lambda f_\lambda(u,v)\varphi_\lambda(x^\mu)$

$$\left(\nabla^2 + m_\lambda^2\right)\varphi_\lambda\left(x^\mu\right) = 0$$

normalization

$$\int dudv \frac{1}{\sin(\alpha_1 + \alpha_2)} f_0(u,v) = 1 \quad \int dudv \frac{1}{\sin(\alpha_1 + \alpha_2)} f_\lambda(u,v) f_{\lambda'}(u,v) = \delta_{\lambda\lambda'}$$

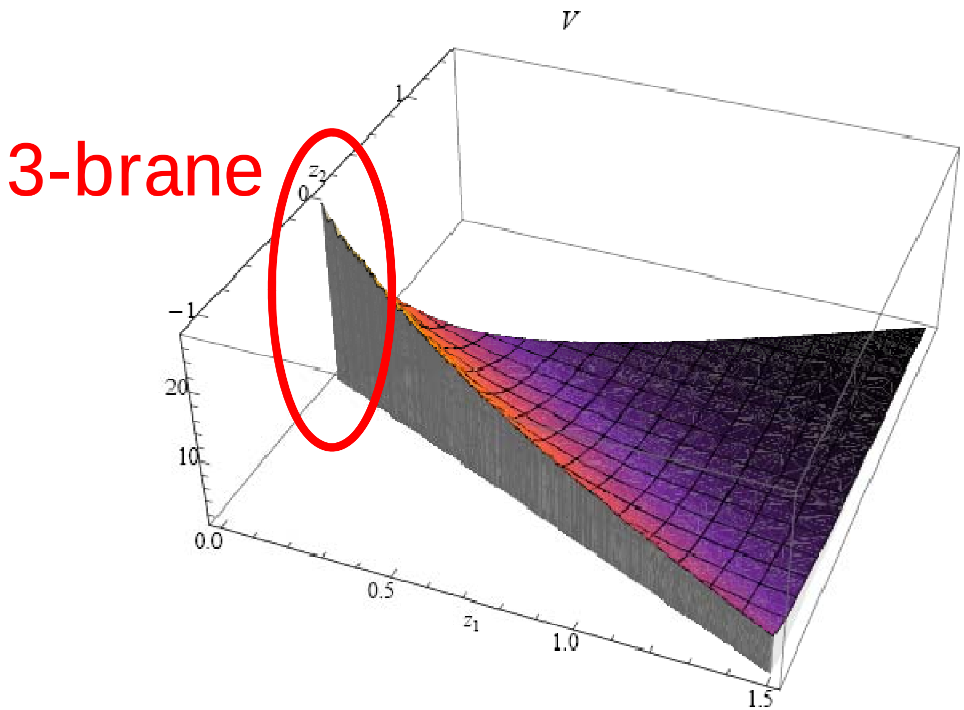

FIG. 1: Scalar field potential V defined in Eq. (11), shown in one of the four patches of AdS_6. The figure illustrates the presence of a potential barrier around the 3-brane. The 3-brane is located at the origin $z_1 = z_2 = 0$ and 4-branes correspond to the boundaries (sides in the plot). Here, we set $\alpha_1 = \pi/5$ and $\alpha_2 = \pi/4$. Note that the behavior of the potential for the vector field V_A defined in Eq. (50) is very similar to the scalar one.

Boundary conditions on 4-branes

$$\left[\left.\frac{\partial}{\partial u}\right|_{u=0+,v} - \left.\frac{\partial}{\partial u}\right|_{u=0-,v} - 2\cos\alpha\, s(v)\left.\frac{\partial}{\partial v}\right|_{u=0,v} + \frac{4(C_1 - C_2\cos\alpha)(1-5\xi)}{C_2|v| + C_0}\right] f = 0$$

$$\left[\left.\frac{\partial}{\partial v}\right|_{v=0+,u} - \left.\frac{\partial}{\partial v}\right|_{v=0-,u} - 2\cos\alpha\, s(u)\left.\frac{\partial}{\partial u}\right|_{v=0,u} + \frac{4(C_2 - C_1\cos\alpha)(1-5\xi)}{C_1|u| + C_0}\right] f = 0$$

Zero mode solutions

$$f_0(u,v) = N_0\left(A^{\alpha_+} + \gamma\, A^{\alpha_-}\right) \quad \alpha_\pm = \frac{1}{2}\left(-1 \pm \sqrt{1 + \frac{4\left(M_s^2 + 30\left(1/5 - \xi\right)\left(k_1^2 + k_2^2\right)\right)}{k_1^2 + k_2^2}}\right)$$

boundary conditions $\xi\left(\xi - \frac{1}{5}\right) - \frac{M_s^2}{100\left(k_1^2 + k_2^2\right)} = 0$

normalizability $1 - \alpha_\pm < 0$ ⇒ $\gamma = 0$

$\xi < \frac{1}{10} \quad \xi > \frac{2}{5}$

Zero mode can be localized

massless, minimally coupled scalar is localized

$M_s = 0,\ \xi = 0$

VECTOR SECTOR

$$S_V = \int d^6X \sqrt{-g}\left(-\frac{1}{4} F_{MN}^2 + \left(\left(M_A^2 - \chi R \right) g^{AB} - \tau R^{AB} \right) A_A A_B \right)$$

mode decomposition $A_\mu = \frac{1}{A} f_0 V_\mu^{(0)} + \int d\lambda \frac{1}{A} f_\lambda V_\mu^{(\lambda)}$

BCs on 4-branes $\frac{M_A^2}{k_1^2 + k_2^2} = 2\tau + (10\chi + \tau)^2$

Localizability (normalizability) condition

$$10\chi + \tau < 0 \quad 10\chi + \tau > 3$$

Massless and minimally coupled vector field cannot be localized

Massless field can be localized for $\tau < 0 \quad \chi < -\frac{\tau}{6}$

FERMIONS

Fermionic perturbations

$$S=\int d^6X\sqrt{-g}\,i\overline{\Psi}\left(\underline{\gamma}^M D_M+M_f\right)\Psi$$

$$\Psi=A^{-5/2}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$ ψ_i Dirac spinors in 4D

$$\underline{\gamma}_M=e_M^A\gamma_A$$ e_M^A : seches-vein

Mass matrices $M_f=\dfrac{1}{A}\begin{pmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{pmatrix}$

$$M_{11}\quad M_{22}\propto I$$

$$M_{12}=\kappa_1\left(-\frac{A_{,z^1}}{A}+i\frac{A_{,z^2}}{A}\right)\qquad M_{21}=\kappa_2\left(\frac{A_{,z^1}}{A}+i\frac{A_{,z^2}}{A}\right)$$

$\kappa_{1,2}$ dimensionless parameters

Mode decomposition

$$\Psi = A^{-5/2}\begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} = \int d\lambda \begin{pmatrix} f_1^{(\lambda)}(z)\psi_1^{(\lambda)}\left(x^\mu\right) \\ f_2^{(\lambda)}(z)\psi_2^{(\lambda)}\left(x^\mu\right)\end{pmatrix}$$

Equation of motion

$$Df_1^{(\lambda)} - M_{21}f_1^{(\lambda)} = im_2^{(\lambda)}f_2^{(\lambda)}, \quad D^* f_2^{(\lambda)} + M_{12}f_2^{(\lambda)} = im_1^{(\lambda)}f_1^{(\lambda)}$$

4D effective theory

$$S = i\int d^4X d\lambda \left(\bar{\psi}_1^{(\lambda)}\gamma^\mu\partial_\mu\psi_1^{(\lambda)} + \bar{\psi}_2^{(\lambda)}\gamma^\mu\partial_\mu\psi_2^{(\lambda)} + im_1^{(\lambda)}\bar{\psi}_1^{(\lambda)}\psi_2^{(\lambda)} + im_2^{(\lambda)}\bar{\psi}_2^{(\lambda)}\psi_1^{(\lambda)} + \bar{\psi}_1^{(\lambda)}M_{11}\psi_1^{(\lambda)} + \bar{\psi}_2^{(\lambda)}M_{22}\psi_2^{(\lambda)}\right)$$

Identifying 4D spinors as a chiral pair of (a)

$$\psi_1^{(\lambda)} := \psi_{L,a}^{(\lambda)} = \frac{1}{2}(1-\gamma_5)\psi_{(a)}^{(\lambda)}, \quad \psi_2^{(\lambda)} := \psi_{R,a}^{(\lambda)} = \frac{1}{2}(1+\gamma_5)\psi_{(a)}^{(\lambda)}$$

The effective action reduces to that of $\psi^{(a)}$

$$S = i\int d\lambda \int d^4x \left(\overline{\psi}_{(\lambda)}^{(a)}\gamma^\mu\partial_\mu\psi_{(\lambda)}^{(a)} + iM_{(\lambda)}^{(a)}\overline{\psi}_{(\lambda)}^{(a)}\psi_{(\lambda)}^{(a)}\right)$$

$$M_{(\lambda)}^{(a)} = m_1^{(\lambda)} + m_2^{(\lambda)}$$

zero mode $M_{(\lambda)}^{(a)} = m_1^{(\lambda)} = m_2^{(\lambda)} = 0$

$$f_1^{(\lambda)} = A^{\kappa_1} \qquad f_2^{(\lambda)} = A^{\kappa_2}$$

localizability condition of each chiral mode $\kappa_i > 1$

$\kappa_1 > 1 \quad \kappa_2 < 1$

Only the left mode is localized.

⇒ Chiral asymmetry

HIGHER MODES

wave function of higher modes

$$
\begin{aligned}
f_\lambda &= \int_0^{q_{\max}} dq \sqrt{X+C_0} \\
&\times \Big\{ a_{1q} \cos\Big[K(X+C_0) - qY\Big] J_\nu\Big(Q_\lambda(q)(X+C_0)\Big) + a_{2q} \sin\Big[K(X+C_0) - qY\Big] J_\nu\Big(Q_\lambda(q)(X+C_0)\Big) \\
&+ b_{1q} \cos\Big[K(X+C_0) - qY\Big] Y_\nu\Big(Q_\lambda(q)(X+C_0)\Big) + b_{2q} \sin\Big[K(X+C_0) - qY\Big] Y_\nu\Big(Q_\lambda(q)(X+C_0)\Big) \Big\} \qquad (58)
\end{aligned}
$$

$$
Q_\lambda(q) := \sqrt{\mu^2 - \frac{4q^2 C_1 C_2 \sin^2\alpha}{k_1^2 + k_2^2}}, \quad K(q) := \frac{C_1^2 - C_2^2}{k_1^2 + k_2^2} q.
$$

$$
\nu := \sqrt{\tilde{M}^2/(k_1^2 + k_2^2) + 1/4}, \; \mu := m_\lambda / \sqrt{k_1^2 + k_2^2}
$$

In general, it is not straightforward to find coefficients a_{iq}, b_{iq}

In massless and minimally coupled case $M_S = 0 \quad \xi = 0 \quad \nu = 5/2$

a_{iq}, b_{iq} are suppressed by factor $m^2 L^2 / M_4$ for smaller q and by factor $1/M_4$ for $q \le q_{\max}$

Higher modes corrections are suppressed by factor M_4

SUMMARY

6D intersecting branes have interesting property

two 4-branes intersecting on our 3-brane in 6D AdS

4D gravity

TeV scale gravity in 6D

A hybrid construction of LED and RS

For appropriate choices of the parameters, all the spin-0, ½, 1 particles are localized at the brane intersection.

reproduction of SM

spectrum at the intersection

Remarks

Massless, mininamally coupled scalar is localized

Vector field must have coupling to the curvature

Chiral asymmetry is naturally realized

Higher modes are unimportant at the intersection

Future prospects

: thin brane ⇒ thick brane

more realistic

curing the codimension-two singularity

more bound states

⇒ may reproduce 3 generations of quark and leptons

: multiply intersecting branes

may give rise to a new localization mechanism

THANK YOU